MathCounts Competitions
(2010)

Chapter Competition
Sprint Round
1. In this Number Wall, you add the numbers next to each other and write the sum in the block directly above the two numbers. Which number will be in the block labeled ‘n’?

\[
\begin{array}{ccc}
46 & & \\
15 & & \\
4 & 8 & 7
\end{array}
\]

1. __________

2. How many integers between 500 and 1000 contain both the digits 3 and 4?

2. __________ integers

3. Giselle will combine blue paint, green paint and white paint in the ratio 3:2:4, respectively. If she uses 12 quarts of white paint, how many quarts of green paint should she use?

3. __________ quarts

4. If $m \boxdot n = (m^2 - n) \div n$, for all real numbers $m$ and $n$, where $n \neq 0$, what is the value of $6 \boxdot 3$?

4. __________

5. How many six-inch by six-inch square tiles are needed to cover a three-foot by two-foot rectangular section of floor?

5. __________ tiles

6. In the circle with center O and diameters AC and BD, the angle AOD measures 54 degrees. What is the measure, in degrees, of angle AOB?

6. __________ degrees
7. A copy of the *Sunday Times* costs $0.95 more than a copy of the *Sunday News*. Ms. Jones buys both papers for $7.25. How much does the *Sunday News* cost?

8. For what value of $m$ will the graphs of the three lines $y = 2x + 1$, $x = 3$, and $y = mx + 3$ have a single point of intersection? Express your answer as a common fraction.

9. The 20 leaves of a 40-page book are numbered 1 through 40. The first and last leaves are ripped out. What is the sum of the remaining page numbers?

10. John opened a new bag of jelly beans and ate three-fourths of the jelly beans in the bag. Then Mike ate two-thirds of the remaining jelly beans. Finally, Fred ate the ten jelly beans that were left. How many jelly beans were in the unopened bag of jelly beans?

11. Joshua has taken four tests. His median score is 80 points and the difference between his greatest score and his least score is 12 points. What is the maximum possible value of the mean of his scores?

12. Regular pentagon ABCDE and regular hexagon AEFGHI are drawn on opposite sides of line segment AE such that they are coplanar. What is the degree measure of angle DEF?
13. The sum of two numbers is 20. If four times the larger number exceeds three times the smaller number by 143, what is the larger number?

14. The first and seventh terms of a sequence are each 10. Starting with the third term, each term is the sum of the previous two terms. What is the fifth term?

15. All of the possible positive four-digit integers are formed that use only the digits 1, 3, 5 and 7 with repetition of digits allowed. How many of these four-digit integers are palindromes?

16. If it takes 96 people 6 days to pave 1 mile of road, how many days will it take 64 people to pave 2 miles of road? (The time required to pave a road is directly proportional to the length of the road and inversely proportional to the number of workers on the job.)

17. A grandfather clock chimes at the start of each hour. Beginning precisely at six o’clock the grandfather clock chimes 6 times with the final chime beginning exactly 10 seconds after six o’clock. If the chimes are uniformly spaced, how long after 12 o’clock does the twelfth chime begin?

18. A bag contains exactly six balls; two are red and four are green. Sam randomly selects one of the six balls and puts it on the table. Then he randomly selects one of the five remaining balls. What is the probability that the two selected balls are of different colors? Express your answer as a common fraction.
19. What is the value of \((1 - \frac{1}{2}) + (2 - \frac{1}{2}) + (3 - \frac{1}{2}) + \ldots + (50 - \frac{1}{2})\)?

20. How many subsets of \(\{1, 2, 3, 4, 5, 6\}\) have either 4 or 5 as their largest element?

21. The average of \(A\) and \(3B\) is 7, the average of \(A\) and \(3C\) is 8, and the average of \(A\) and \(3D\) is 9. What is the average of \(A, B, C\) and \(D\)?

22. Ms. Quinn traveled at a constant speed for a distance of 50 miles on her trip to a campground. On a later trip, she traveled at a constant speed for a distance of 400 miles to a ski resort, driving three times as fast as she had on her earlier trip to the campground. What is the ratio of the time it took for her second trip to the time it took for her first trip? Express your answer as a common fraction.

23. In rectangle \(ABCD\), side \(AB\) measures 6 units and side \(BC\) measures 3 units, as shown. Points \(F\) and \(G\) are on side \(CD\) with segment \(DF\) measuring 1 unit and segment \(GC\) measuring 2 units, and lines \(AF\) and \(BG\) intersect at \(E\). What is the area of triangle \(AEB\)?

24. In a survey of 100 readers, all of whom liked fiction and/or non-fiction, three times as many people reported liking fiction as those who reported liking non-fiction. Of those surveyed, eight liked both fiction and non-fiction. How many people reported liking non-fiction?
25. An equilateral triangle has two vertices at (0, 5) and (8, 5). If the third vertex is in the first quadrant, what is its $y$-coordinate? Express your answer in simplest radical form.

26. What is the area enclosed by the graph of $|x| + |y| = 10$?

27. If $f(x) = 5x - 3$, $g(x) = 3x^2 + 1$ and $h(x) = f(x) + g(x)$, what is the sum of the $x$-values for which $h(x) = 0$? Express your answer as a common fraction.

28. Two real numbers have an average of 7. The average of their squares is 54. What is the product of the two numbers?

29. In triangle ABC, AB = AC and D is a point on $\overline{AC}$ so that $\overline{BD}$ bisects angle ABC. If $\overline{BD} = \overline{BC}$, what is the measure, in degrees, of angle A?

30. Given $m$ and $n$ are positive integers such that for some positive integer $x$, $\sqrt{x+43} = m$ and $\sqrt{x+16} = n$, what is the value of the product $mn$?