MathCounts Competitions (2009)

Chapter Competition
Sprint Round
1. In the integer 45,075,123, the 2 represents the value 20. By what factor would the value represented by the 5 in the thousands place have to be multiplied to equal the value represented by the 5 in the millions place?

2. Twenty-seven increased by twice a number is 39. What is the number?

3. Using the bar graph, what is the positive difference between number of students at the school with the largest enrollment and the number of students at the school with the smallest enrollment?

4. June and Julia live 1 mile apart. It takes June 4 minutes to ride her bike directly to Julia's house. At the same rate, how long would it take June to ride the 3.5 miles from her own house to Bernard's house?

5. Two complete cycles of a pattern are shown below. If the pattern continues, what will be the 103rd letter?

AABBBCCCSCCCCAABBBCCCSCC...
6. What is the length, in inches, of segment AB? Express your answer as a decimal to the nearest hundredth.

6. ______ inches

7. What is the positive difference between the sum of $\frac{1}{2}$ and $\frac{4}{1}$ and the product of $\frac{1}{2}$ and $\frac{1}{7}$? Express your answer as a common fraction.

7. ______

8. If three identical cylinders weigh as much as five identical spheres, and three of these spheres weigh as much as twelve identical cubes, then how many cylinders weigh as much as sixty cubes?

8. ______ cylinders

9. Bill and Jill both exercise on Monday, January 1. Bill exercises every 5th day and Jill exercises every 4th day. What day of the week (Sunday, Monday, Tuesday, etc.) is the next day on which they both exercise?

9. ______

10. Rayna writes each letter of her name—R, A, Y, N and A—on a separate piece of 1-by-1 paper. She then puts the 5 pieces of paper in a bag and randomly selects one of the 5 pieces. What is the probability that she selects the piece with the R written on it? Express your answer as a common fraction.

10. ______

11. Amanda, Ben and Carlos share a sum of money. Their portions are in the ratio of 1:2:7, respectively. If Amanda’s portion is $20, what is the total amount of money shared?

11. $_________
12. If \( \frac{x}{5} \leq \frac{2}{x} \) and \( x \) is a positive integer, how many distinct values are possible for \( x \)?

13. Louis earns a base monthly salary of $1200 with 5% commission on sales. For a month with $25,000 in sales, what are Louis’ total earnings?

14. Set L contains exactly four consecutive, positive, odd integers. The sum of the greatest integer and twice the least integer is 39. What is the least integer in the set?

15. Janea has a square garden with an area of 1600 square feet. She plans to create a straight path from the middle of one side to the middle of the opposite side. If the path is 3 feet wide, how many square feet of garden will be turned into path?

16. For integers \( a, b \) and \( k \), we know that \( a > 12, b < 20 \) and \( a < b \). If \( b = 7k \), what is the value of \( k \)?

17. A fraction, \( \frac{a}{b} \), not necessarily in lowest terms is equivalent to \( \frac{2}{5} \). When 5 is added to the denominator of \( \frac{a}{b} \), the value of the new fraction becomes equivalent to \( \frac{1}{2} \). What is the sum of \( a \) and \( b \)?
18. How many positive whole-number divisors does 196 have?

19. Lines $l$ and $m$ are parallel and points $A$, $B$ and $D$ are collinear. What is the value of $x$?

20. What is the only integer value of $n$ for which $\frac{n+1}{13-n}$ is a positive prime number?

21. A set of 7 positive integers has a unique mode of 1, a mean of 5 and a median of 6. What is the largest possible value for any of the integers in the set?

22. Given that the diagonals of a rhombus are perpendicular bisectors of each other, what is the perimeter of a rhombus with diagonals of lengths 12 and 16 units?

23. Chun is playing a game in which he can score 0, 2, 6 or 10 points in each round. After four rounds the sum of his scores is 16. How many different scoring sequences could have produced this sum? (One such sequence to include is 0, 6, 10, 0.)

24. Janice bought 30 items each priced at 30 cents, 2 dollars or 3 dollars. If her total purchase price was $30.00, how many 30-cent items did she purchase?
25. If the ratio of $2x - y$ to $x + y$ is 2 to 3, what is the ratio of $x$ to $y$? Express your answer as a common fraction.

26. The members of a band are arranged in a rectangular formation. When they are arranged in 8 rows, there are 2 positions unoccupied in the formation. When they are arranged in 9 rows, there are 3 positions unoccupied. How many members are in the band if the membership is between 100 and 200?

27. Jerry is presently twice as old as his brother and six years older than his sister. How many years from now will Jerry's age be $\frac{2}{3}$ of the combined ages of his brother and sister at that time?

28. In a sequence of positive integers each term after the first is $\frac{1}{3}$ of the sum of the term that precedes it and the term that follows it in the sequence. What is the 5th term of this sequence if the 1st term is 2 and the 4th term is 34?

29. In a 7-by-7 checkerboard, as shown, two unit squares will be chosen at random and without replacement. What is the probability that the two squares are adjacent to each other (share a side)? Express your answer as a common fraction.

30. A circle is centered at (5, 15) and has a radius of $\sqrt{130}$ units. Point Q(x, y) is on the circle, has integer coordinates, and the value of the x-coordinate is twice the value of the y-coordinate. What is the maximum possible value for x?
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<tbody>
<tr>
<td>1.</td>
<td>1000</td>
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<td>2.</td>
<td>6</td>
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<td>3.</td>
<td>650 students</td>
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<td>4.</td>
<td>14 minutes</td>
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<tr>
<td>5.</td>
<td>B</td>
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<td>6.</td>
<td>3.25 inches</td>
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<td>7.</td>
<td>$\frac{2}{3}$</td>
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<td>8.</td>
<td>9 cylinders</td>
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<td>9.</td>
<td>Sunday</td>
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<td>10.</td>
<td>$\frac{1}{5}$</td>
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<td>11.</td>
<td>$$200$ or $200.00$</td>
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<td>12.</td>
<td>3 values</td>
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<td>13.</td>
<td>$$2450$ or $2450.00$</td>
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<tr>
<td>14.</td>
<td>11</td>
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<tr>
<td>15.</td>
<td>120 sq feet</td>
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<td>16.</td>
<td>2</td>
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<td>17.</td>
<td>70</td>
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Sprint Round

18. 9 divisors

19.  

20. 12 

21. 13 

22. 40 units 

23. 22 sequences 

24. 20 items 

25. \( \frac{5}{4} \)

26. 150 members

27. 12 years

28. 89 

29. \( \frac{1}{14} \)

30. 12