Mathcounts Preparation

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1. What is the sum of the first 1000 integers? Extension: First 1000 odd integers? 1000 even?
   Concept #1

2. How many ordered triples \((x, y, z)\) of positive integers have the property that \(x + y + z = 6\).
   Concept #2

3. How many different combinations of three numbers can be selected from the set \(\{1, 2, 3, 4, 5, 6.7, 8\}\) so that the numbers could represent the side lengths of a triangle?
   Concept #3

4. What is the least positive integer \(n\) such that the product \(42n\) is a perfect square?

5. What is the sum of the first 200 terms of the arithmetic sequence \(2, 5, 8, 11, 14\ldots\)?

6. What is the sum of the infinite geometric sequence: \(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\)?
   Concept #4

7. What is the largest area that can be enclosed with 40 feet of fencing?

8. Thomas, John, and Patrick are the only people registered in a math competition, and they have \(1/3\), \(1/6\), and \(1/2\) chances of winning, respectively. The night before the competition, John is diagnosed with Ebola, and is disqualified from the tournament. After the tragedy, what is the probability Thomas will win?

9. Terry and Big Benjamin work together to finish a 5000 calorie meal. Terry is recovering from anorexia, and would take 5 hours to finish the meal by herself. Fortunately, Big Benjamin, a professional eater, would take about 30 minutes to finish the meal by himself. How many hours will it take the two working together to finish the meal?

10. Martha is a track runner and consistently runs at 8 miles per hour. One day, she decided to go out for a short run to her friend’s house, which was 500 miles away. Simultaneously, Martha’s mother George received an email from the school principal about Martha’s poor grades. After donating all of Martha’s possessions to the local Goodwill, George departed 5 hours later following Martha’s tracks and running at a speed of 16 miles per hour. How many hours after George’s departure, will she catch up to Martha?
Concepts:

1. The sum of the first \( n \) counting numbers is equal to \( \frac{n(n+1)}{2} \).
   a. The sum of the first \( n \) odd counting numbers is equal to \( n^2 \).
      Extension: Try to figure out how the formulas our derived

2. The number of combinations where \( x \) positive numbers sum to \( n \) is equal to \( \binom{n-1}{x-1} \).
   Note: In problem 2, \( n = 6 \) and \( x = 3 \).

3. In any triangle, the sum of the smaller two side lengths is always greater than the largest side length.
   Note: 1,1,2, 2,4,8, and 3,6, 15 would not be triangle side lengths, but 1,4,4, 4,5,7, 15,15 23, would be triangle side lengths.

4. The sum of an infinite geometric series is equal to \( a/(1 - r) \) where \( a \) is the first term and \( r \) is the common ratio.